Chapter 19
Waves in Residual-Saturated Porous Media

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Abstract We present a three-phase model describing wave propagation phenomena in residual-saturated porous media. The model consists of a continuous non-wetting phase and a discontinuous wetting phase and is an extension of classical biphasic (Biot-type) models. The model includes resonance effects of single liquid bridges or liquid clusters with miscellaneous eigenfrequencies taking into account a visco-elastic restoring force (pinned oscillations and/or sliding motion of the contact line). For the quasi-static limit case, i.e., $\omega \mapsto 0$, the results of the model are identical with the phase velocity obtained with the well-known Gassmann–Wood limit.

19.1 Introduction and Motivation

Understanding the dynamical and acoustical behavior of porous rocks is of great importance in geophysics, e.g., earthquakes, and for various seismic applications, e.g., hydrocarbon exploration. While many studies investigated wave propagation in fully-saturated porous rocks analytically and numerically, cf. [13, 5–7, 27], studies for partially-saturated porous rocks are rare, cf. [8, 20, 14, 4, 23, 24, 26, 29, 31, 32, 9, 18, 19]. However, several physical processes relevant at low frequencies take place only in partially saturated rocks but not in fully saturated rocks such as, for

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example, capillarity-induced resonance of oil blobs [16] or wave-induced fluid flow (cf. [33, 11]). Capillarity-induced resonance of trapped oil blobs at residual saturation can be excited by sound waves and is caused by the contact-line dynamics [16, 15]. The resonant oscillations of fluid blobs can also cause considerable attenuation at the resonance frequency [2]. However, in models describing such resonance effects the porous skeleton is often assumed rigid and only relative movements between the discontinuous fluid blobs and the solid skeleton are considered [3, 15, 16]. Reference [12] presented a first attempt to combine the dynamical behavior of wave propagation and rock-internal oscillations caused by capillarity effects.

Wave-induced fluid flow in partially-saturated rocks is considered to be an important wave attenuation mechanism at low frequencies where the viscous (i.e., dissipative) flow is caused by the differences in pore fluid pressures, cf. [33, 11, 22, 28]. Wave-induced flow caused by partial saturation is frequently studied with the so-called patchy-saturation models that apply Biot's theory (i.e., a two-phase model for full saturation) with spatially varying material parameters, for example, fluid compressibilities, to model a partial saturation on the meso-scale, i.e., scale larger than the pore size but smaller than the wavelength, cf. [33, 11]. Such models have attracted considerable attention because they predict significant attenuation and dispersion for low frequencies in wide agreement with laboratory measurements [21] and field observations [10]. However, applying the theory for fully-saturated rocks to study partially-saturated rocks causes some disadvantages such as: (i) the saturation is not a primary model parameter, (ii) partial saturation can only be modeled on the meso-scale but not on the micro-scale, (iii) the calculation of the effective meso-scale patch size distribution from the real continuous saturation distribution is not obvious, and (iv) the impact of capillary pressure is neglected which may be especially important for residual saturations (e.g., [7] speculated that capillary pressure could have a significant influence on phase velocities and attenuation).

In the present investigation, our aim is to study attenuation due to fluid oscillations and attenuation due to wave-induced flow with the macroscopic three-phase model, i.e., a mixture consisting of one solid constituent building the elastic skeleton and two immiscible fluid constituents. Thus we briefly describe the governing field equations, especially the momentum interaction between the inherent constituents. Finally, we study monochromatic waves in transversal and longitudinal direction and discuss the resulting dispersion relations for a typical reservoir sandstone equivalent (Berea sandstone).

19.2 Field Equations of the Multi-Phase Model

In a certain Representative Volume Element (RVE) with volume $\mathrm{d}v$, the volume fractions of the constituents φ^α are defined as $n^\alpha = \mathrm{d}v^\alpha/\mathrm{d}v$. The volume occupied by a single constituent φ^α is defined as $\mathrm{d}v^\alpha$. Furthermore, the partial densities are introduced as the mass of the constituent $\mathrm{d}m^\alpha$ related to the volume of the RVE $\mathrm{d}v$, i.e., $\rho^\alpha := \mathrm{d}m^\alpha/\mathrm{d}v$. The true densities are given by $\rho^{\alpha R} := \mathrm{d}m^\alpha/\mathrm{d}v^\alpha$. Note

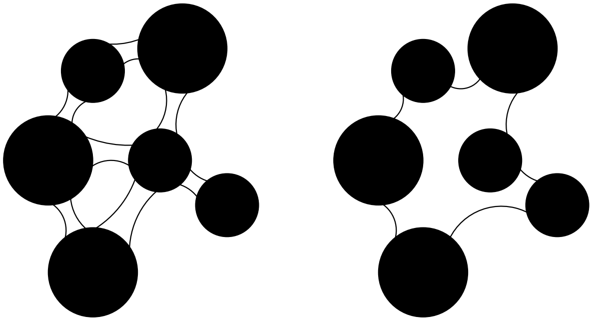


Fig. 19.1 A residual-saturated porous medium. (*Left*) Pendular state with individual liquid bridges. (*Right*) Funicular state with a liquid cluster, cf. [1, 25]

that both densities are related by the volume fractions, i.e., $\rho^\alpha = n^\alpha \rho^{\alpha R}$. Let us denote the continuous phases as φ^β with $\beta \in \{\mathfrak{s}, \mathfrak{n}\}$ ($\mathfrak{s}$ denoting the solid phase and $\mathfrak{n}$ denoting the non-wetting phase) while the discontinuous wetting fluid phase is denoted as $\varphi^\mathfrak{w}$. On the one hand, the geometry and mass distribution of the discontinuous phase is rather complex in realistic porous media. On the other hand, this information, obtained, e.g., by modern non-destructive imaging techniques, in principle allows calculating the eigenmodes ω_i of the liquid blobs numerically, cf. Fig. 19.1. This topic will be investigated in detail in the future. An obvious result from the realistic, i.e., inhomogeneous distribution of the discontinuous liquid in the pore space of the material is that liquid bridges or clusters with varying eigenfrequencies ω_i have to be taken into account. From a modeling perspective, this could be, for instance, captured with a probability distribution function in the RVE. We begin by formulating the partial density $\rho_i^\mathfrak{w}$ of a single blob $\varphi_i^\mathfrak{w}$ with a certain eigenfrequency ω_i. Furthermore, we can introduce the partial density of the total wetting phase $\rho^\mathfrak{w}$ in the RVE

$$\rho_i^\mathfrak{w} = \frac{z_i \, \mathrm{d}m_i^\mathfrak{w}}{\mathrm{d}v}, \quad \text{and} \quad \rho^\mathfrak{w} = \sum_{i=1}^{z} \rho_i^\mathfrak{w} = \rho^{\mathfrak{w}R} \sum_{i=1}^{z} n_i^\mathfrak{w}. \tag{19.1}$$

In contrast to a continuous distribution of eigenfrequencies in the RVE, we focus ourselves on a set of discrete eigenfrequencies. Note that the number of blobs in the RVE with the eigenfrequency ω_i is given by z_i, and z is the total number of (discrete) eigenfrequencies.

19.2.1 Balance Equations of Momentum of Continuous Phases

The balance equations of momentum in local form for a biphasic mixture consisting of a solid skeleton $\varphi^\mathfrak{s}$ and a continuous non-wetting phase $\varphi^\mathfrak{n}$ are written in the

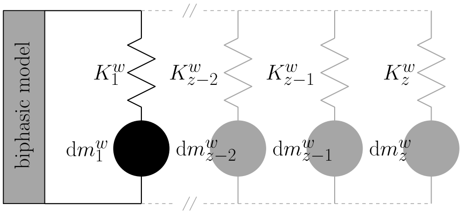


Fig. 19.2 Rheology of a residual-saturated porous model with a coupled set of z undamped oscillators with eigenfrequency $\omega_i(K_i^z, \mathrm{d}m_i^{\mathfrak{w}})$. Note that each oscillator could occur z_i-time

following form

$$\rho^{\mathfrak{s}}\ddot{\mathbf{u}}_{\mathfrak{s}} - \operatorname{div}\boldsymbol{\sigma}^{\mathfrak{s}} = \hat{\mathbf{p}}^{\mathfrak{s}}, \quad \text{and} \quad \rho^{\mathfrak{n}}\ddot{\mathbf{u}}_{\mathfrak{n}} - \operatorname{div}\boldsymbol{\sigma}^{\mathfrak{n}} = \hat{\mathbf{p}}^{\mathfrak{n}}. \tag{19.2}$$

Furthermore, we assume that the wetting phase $\varphi^{\mathfrak{w}}$ is in a discontinuous state of residual saturation. Thus, the momentum interaction of the solid constituent consists of a viscous (Darcy-like) term and an viscoelastic restoring force caused by the oscillations of the wetting phase. The viscous momentum term describes the exchange of (non-equilibrium) momentum between the solid and the non-wetting phase while the elastic momentum term captures the exchange of equilibrium momentum between the solid and the pinned discontinuous wetting phase. With respect to the large differences in compressibilities between an (incompressible) wetting fluid (oil, water) and a (compressible) non-wetting fluid (gas), we are able to neglect the exchange of momentum between the fluid phases.

$$\sum_\alpha \hat{\mathbf{p}}^\alpha = \mathbf{0}, \qquad \hat{\mathbf{p}}^{\mathfrak{s}} = -\big(\hat{\mathbf{p}}^{\mathfrak{n}} + \hat{\mathbf{p}}^{\mathfrak{w}}\big), \quad \text{and} \quad \hat{\mathbf{p}}^{\mathfrak{w}} = \sum_i \hat{\mathbf{p}}_i^{\mathfrak{w}}. \tag{19.3}$$

19.2.2 Balance Equation of Momentum of Discontinuous Wetting Phase

As we would like to describe pinned oscillations as well as sliding motion of the wetting constituent, the balance of momentum for one single liquid patch is given by a damped oscillator-like equation (Fig. 19.2)

$$\mathrm{d}m_i^{\mathfrak{w}}\ddot{\mathbf{u}}_{\mathfrak{w}}^i = -\mathrm{d}m_i^{\mathfrak{w}}\omega_i^2\big(\mathbf{u}_{\mathfrak{w}}^i - \mathbf{u}_{\mathfrak{s}}\big) - \mathrm{d}c_i^{\mathfrak{w}}\big(\dot{\mathbf{u}}_{\mathfrak{w}}^i - \dot{\mathbf{u}}_{\mathfrak{s}}\big) = \mathbf{f}_i^{\mathfrak{w}}. \tag{19.4}$$

This equation can be easily averaged in the volumetrical sense within the RVE. In the RVE, we obtain the coarse-grained equation for one eigenfrequency ω_i

$$\rho_i^{\mathfrak{w}}\ddot{\mathbf{u}}_{\mathfrak{w}}^i = -\rho_i^{\mathfrak{w}}\omega_i^2\big(\mathbf{u}_{\mathfrak{w}}^i - \mathbf{u}_{\mathfrak{s}}\big) - c_i\big(\dot{\mathbf{u}}_{\mathfrak{w}}^i - \dot{\mathbf{u}}_{\mathfrak{s}}\big) = \hat{\mathbf{p}}_i^{\mathfrak{w}}. \tag{19.5}$$

An equation equivalent to (19.5) exists for each set of oscillators with a certain eigenfrequency ω_i. According to classical results obtained from equilibrium and non-equilibrium evaluation of the balance of entropy, we obtain results for the momentum interaction terms $\hat{\mathbf{p}}^\alpha = \hat{\mathbf{p}}^\alpha_{\text{eq}} + \hat{\mathbf{p}}^\alpha_{\text{neq}}$ of the constituents:

$$
\begin{aligned}
&\hat{\mathbf{p}}^{\mathfrak{n}}_{\text{eq}} = p \,\text{grad}\, n^{\mathfrak{n}} \quad \text{and} \quad \hat{\mathbf{p}}^{\mathfrak{n}}_{\text{neq}} = -b_0\big(\dot{\mathbf{u}}_{\mathfrak{n}} - \dot{\mathbf{u}}_{\mathfrak{s}}\big),\\
&\hat{\mathbf{p}}^{\mathfrak{w}}_{\text{eq}} = p \,\text{grad}\, n^{\mathfrak{w}} \quad \text{and}\\
&\quad \hat{\mathbf{p}}^{\mathfrak{w}}_{\text{neq}} = -\sum_i \big[\rho^{\mathfrak{w}}_i \omega_i^2 \big(\mathbf{u}^i_{\mathfrak{w}} - \mathbf{u}_{\mathfrak{s}}\big)\big] - \sum_i \big[c_i\big(\dot{\mathbf{u}}^i_{\mathfrak{w}} - \dot{\mathbf{u}}_{\mathfrak{s}}\big)\big],\\
&\hat{\mathbf{p}}^{\mathfrak{s}}_{\text{eq}} = p \,\text{grad}\, n^{\mathfrak{s}} \quad \text{and} \quad \hat{\mathbf{p}}^{\mathfrak{s}}_{\text{neq}} = \hat{\mathbf{p}}^{\mathfrak{n}}_{\text{neq}} + \hat{\mathbf{p}}^{\mathfrak{w}}_{\text{neq}},
\end{aligned}
\tag{19.6}
$$

with $b_0 = [(n^{\mathfrak{n}})^2 \gamma^{\mathfrak{g}R}]/k^{\mathfrak{n}}$. Note that Eq. (19.6)b refers to the wetting constituent, i.e., the sum of all damped oscillators. Furthermore, we have introduced the effective weight of the non-wetting fluid $\gamma^{\mathfrak{n}R}$ and the Darcy permeability (hydraulic conductivity) $k^{\mathfrak{n}}$. Inserting stress-strain relations (Hooke's law) and neglecting the convective part of the material time derivatives (linearized model), we obtain the linear set of field equations exemplarily written down for the special case of a set of oscillators with two distinct eigenfrequencies ω_1 and ω_2

$$
\begin{aligned}
N \,\text{div}\,\text{grad}\, \mathbf{u}_{\mathfrak{s}} + (A+N)\,\text{grad}\,\text{div}\, \mathbf{u}_{\mathfrak{s}} + Q \,\text{grad}\,\text{div}\, \mathbf{u}_{\mathfrak{n}} &= \rho^{\mathfrak{s}} \ddot{\mathbf{u}}_{\mathfrak{s}} - \hat{\mathbf{p}}^{\mathfrak{s}}_{\text{neq}},\\
Q \,\text{grad}\,\text{div}\, \mathbf{u}_{\mathfrak{s}} + R \,\text{grad}\,\text{div}\, \mathbf{u}_{\mathfrak{n}} &= \rho^{\mathfrak{n}} \ddot{\mathbf{u}}_{\mathfrak{n}} - \hat{\mathbf{p}}^{\mathfrak{n}}_{\text{neq}},\\
0 &= \rho^{\mathfrak{w}}_1 \ddot{\mathbf{u}}^1_{\mathfrak{w}} - \hat{\mathbf{p}}^{\mathfrak{w}}_{1,\text{neq}},\\
0 &= \rho^{\mathfrak{w}}_2 \ddot{\mathbf{u}}^2_{\mathfrak{w}} - \hat{\mathbf{p}}^{\mathfrak{w}}_{2,\text{neq}}.
\end{aligned}
\tag{19.7}
$$

The set of field equations (19.7) can be regarded is an extension of the classical Biot equations, cf. [5–7]. The bi-phasic limit case given by $n^{\mathfrak{w}} = 0$ reduces (19.7) to two the Biot equations (19.7)a,b. The parameters $P := 2N + A$, Q and R are the classical Biot parameters. They can be related to the physical-based shear modulus of the solid skeleton (G) and the bulk moduli of the grains ($K^{\mathfrak{s}}$), the skeleton (K) and the non-wetting fluid ($K^{\mathfrak{n}}$), cf. [26].

19.2.3 Monochromatic Waves

With the help of a standard ansatz for harmonic waves and usual splitting techniques, we study monochromatic waves for the developed three-phase model. For notational convenience, we restrict ourselves again to the special case of a set of oscillators with ω_1 and ω_2.

19.2.3.1 Transversal Mode

The generalized eigenvalue problem for shear waves can be written in matrix notation as $\det(\mathbf{A} - k^2 \mathbf{B}_T) = 0$ with

$$
\mathbf{A} = \begin{bmatrix} \tilde{\rho}^{\mathfrak{s}} - \sum(\frac{\omega_i^2}{\omega^2}) - \sum(\frac{ic_i}{\omega}) & \tilde{\rho}^{\mathfrak{s}\,\mathfrak{n}} & \frac{\omega_1^2}{\omega^2}\tilde{\rho}_1^{\mathfrak{w}} + \frac{ic_1}{\omega} & \frac{\omega_2^2}{\omega^2}\tilde{\rho}_2^{\mathfrak{w}} + \frac{ic_2}{\omega} \\ \tilde{\rho}^{\mathfrak{s}\,\mathfrak{n}} & \tilde{\rho}^{\mathfrak{n}} & 0 & 0 \\ \frac{\omega_1^2}{\omega^2}\rho_1^{\mathfrak{w}} + \frac{ic_1}{\omega} & 0 & \rho_1^{\mathfrak{w}}(1 - \frac{\omega_1^2}{\omega^2} - \frac{ic_1}{\omega\rho_1^{\mathfrak{w}}}) & 0 \\ \frac{\omega_2^2}{\omega^2}\rho_2^{\mathfrak{w}} + \frac{ic_2}{\omega} & 0 & 0 & \rho_2^{\mathfrak{w}}(1 - \frac{\omega_2^2}{\omega^2} - \frac{ic_2}{\omega\rho_2^{\mathfrak{w}}}) \end{bmatrix} \omega^2 \tag{19.8}
$$

and

$$
\mathbf{B}_T = \begin{bmatrix} N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{aligned} \tilde{\rho}^{\mathfrak{s}} &= \rho^{\mathfrak{s}} - ib_0/\omega, \\ \tilde{\rho}^{\mathfrak{n}} &= \rho^{\mathfrak{n}} - ib_0/\omega, \\ \tilde{\rho}^{\mathfrak{s}\mathfrak{n}} &= ib_0/\omega. \end{aligned} \tag{19.9}
$$

The structure of the eigenvalue problems for both cases (transversal and shear mode) clearly shows that the dimension of the characteristic polynomial is of the same order as for the Biot case. Therefore, additional and higher order wave modes cannot be expected.

19.2.3.2 Longitudinal Mode

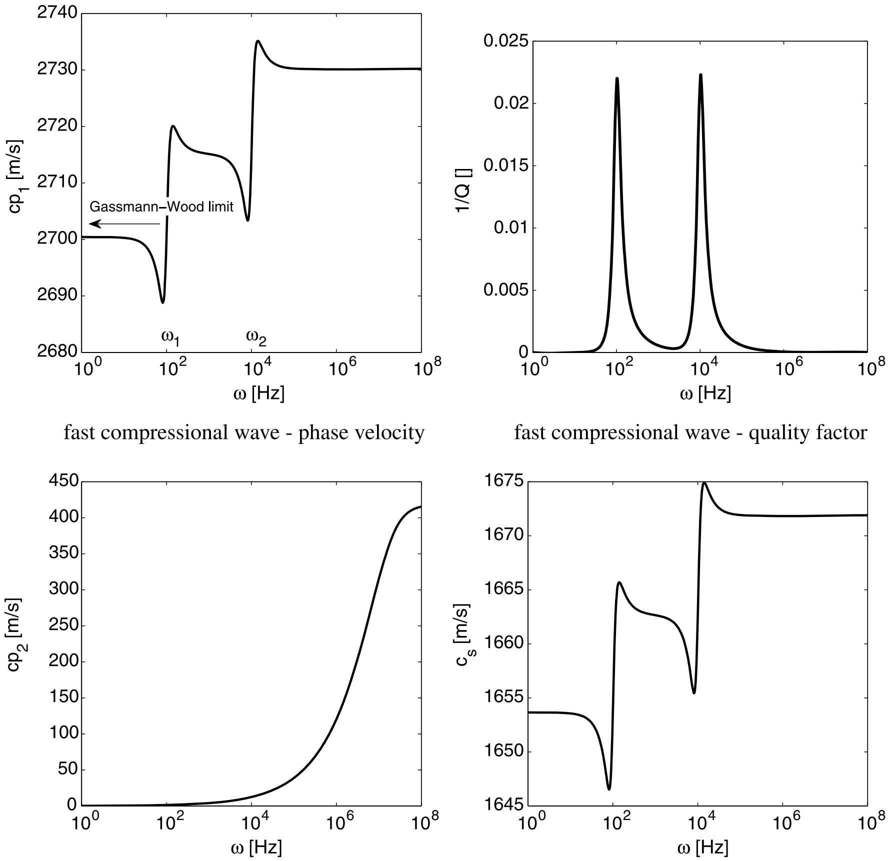
The generalized eigenvalue problem for compressional waves can be written in matrix notation as $\det(\mathbf{A} - k^2\mathbf{B}_L) = 0$. Note that the matrix $\mathbf{A}$ is equal to the case of shear waves.

$$
\mathbf{B}_L = \begin{bmatrix} P & Q & 0 & 0 \\ Q & R & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{19.10}
$$

In Fig. 19.3 we plot the dispersion relations for a residual-saturated Berea sandstone, i.e., a typical reservoir sandstone equivalent.

19.3 Discussion and Conclusion

We propose a new model capturing resonance effects in partially-saturated porous media. The basic rheology of the material consists of the classical Biot mechanism (or physics introduced by equivalent mixture theory-based models) and a set of undamped oscillators with particular eigenfrequencies. The model contains all the effects described by the Biot model, e.g., the existence of a second compressional wave (i.e., Biot slow wave) or the characteristic velocity dispersion of the different waves. Additionally, the presented model shows a distinct dispersive effect governed by the discrete oscillating fluid-blobs or liquid clusters.



fast compressional wave - phase velocity

fast compressional wave - quality factor

slow compressional wave - phase velocity

shear wave - phase velocity

Fig. 19.3 Dispersion relations for a residual-saturated Berea sandstone. According to [30], we assume the following material properties: $G = 6.0$ GPa, $K = 8.0$ GPa, $K^{\mathfrak{s}} = 36.0$ GPa, $k^{\mathfrak{s}} = 190$ mD, $K^{\mathfrak{n}} = 131$ kPa, $\phi_0 = 1 - n^{\mathfrak{s}} = 0.19$ and the saturation of the wetting fluid is $s^{\mathfrak{w}} = 0.25$. Furthermore, the non-wetting fluid has the properties of water and the wetting fluid has the properties of air. The wetting fluid consists of two distinct patch sizes with eigenfrequencies of $\omega_1 = 100$ Hz and $\omega_s = 10$ kHz. The viscous damping parameters c_1 and c_2 are chosen arbitrarily

The proposed model, in its current form, is valid for a discontinuous wetting phase and a continuous non-wetting phase with a much smaller compressibility. One example for such a system is residual water in an otherwise air-filled porous rock or soil, as it occurs, for example, in the vadose zone above the groundwater table. In future studies, we will seek for a more general model that can handle two immiscible fluids with similar compressibilities. With such a model we will be able to study, for example, oscillation effects in hydrocarbon reservoir rocks partially saturated by oil and water. This will be particularly interesting for the case of residual oil saturation in a water-flooded reservoir.

One yet unsolved problem of the current model is the determination of the resonance frequency of individual fluid blobs, fluid bridges or fluid patches. References [16] or [17] give analytical solutions for the resonance frequency of very particular geometries of the fluid blobs. However, it is questionable if these geometries are realistic in real partially saturated rocks. In the presented study, we assume only two discrete values for the eigenfrequency, representing two different sizes of fluid blobs or patches. Eigenfrequencies of naturally occurring fluid blobs or patches will need to be measured in the laboratory or determined by numerical analysis techniques. We expect a certain range of eigenfrequencies for natural geometries, rather than discrete values. However, such eigenfrequency distributions will be relatively easy to implement in the current model.

A second, yet unsolved, problem is the determination of the viscous damping force acting on the individual oscillators. For the presented study, we arbitrarily choose two values, one for each eigenfrequency. However, for future studies, we will try to present a physically-based derivation of the viscous damping force.

Despite the discussed problems, we believe that the newly presented model is an important extension of the Biot model. We include a micro-scale effect, i.e., oscillation of the residual fluid phase, that was not considered in these types of models until now. The presented model contains all the effect described by the Biot model, matches the high- and low-frequency limits of the Biot model and reduces to the original Biot model when the newly introduced parameters are set accordingly. The new model therefore represents the possibility to study micro-scale oscillation effects using a well-known and accepted theory.

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