Theoretical and numerical modeling of waves in three-phase media

-a snapshot of the work in progress -

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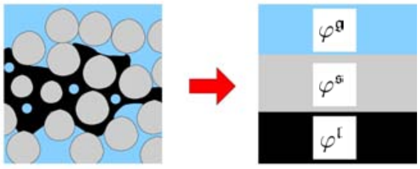
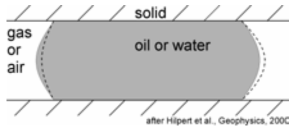
Abstract
We present a mathematical model for wave propagation in a partially saturated porous medium based on the Theory of Porous Media (TPM, e.g. [1, 10]). The medium is composed of a elastic skeleton, a compressible wetting fluid and a compressible non-wetting fluid. The solid grains are assumed incompressible (rigid grain assumption). The capillary pressure depending on saturation is described by the empirical Brooks and Corey equation [6]. Both porosity and saturation depend on the volumetric deformation of all phases and are variable (i.e. they are dependent field variables). We developed a new three-phase model because we want to extend the continuum model and implement the effects of surface tension. The restoring force generated by surface tension can cause oscillations of water or oil blobs in partially-saturated porous material with gas or air as third phase. The restoring force due to surface tension is usually not considered in continuum three-phase models for wave propagation.

The main aims of our study are:
- Coupling wave propagation in partially saturated rocks with pore fluid oscillations caused by surface tension.
- Implementation of attenuation due to fluid oscillations.
- Study attenuation of P- and S-waves in three-phase media due to wave induce flow (e.g., depth dependence due to gas pressure increase).
- Accurate implementation of capillary pressure for cases of very high or low saturation.
- Study scattering of waves by three-phase media.
- Develop a simpler but still accurate three-phase model for partially gas-saturated rocks with respect to existing models [e.g., 3 and 5].



Concept of phenomenological continuum-mixture theory. Individual fluid surfaces disappear during the superposition of the three continuum fields.



Sketch of oil blob oscillations with pinned contact lines. Solid lines: equilibrium state. Dashed lines: excited state. After [8]

Some basic equations of the mathematical model with effective surface tension term

Field equations

$$\rho^s \ddot{\mathbf{u}}_s - div\left(\mathbf{T}_E^s - n^s p\mathbf{I}\right) = -\hat{\mathbf{p}}^f + \rho^s \mathbf{g}$$

$$\rho^l \ddot{\mathbf{u}}_l + grad\left(n^l p^{lR}\right) = \hat{\mathbf{p}}^l + \rho^l \mathbf{g}$$

$$\rho^g \ddot{\mathbf{u}}_g + grad\left(n^g p^{gR}\right) = \hat{\mathbf{p}}^g + \rho^g \mathbf{g}$$

$$\hat{\mathbf{p}}^f = \hat{\mathbf{p}}^f_{eq} + \hat{\mathbf{p}}^f_{neq} = n^l grad\left(p^{lR}\right) + n^g grad\left(p^{gR}\right) - \frac{\left(n^l\right)^2 \gamma^{lR}}{k^{lR}} \mathbf{w}_l - \frac{\left(n^g\right)^2 \gamma^{gR}}{k^{gR}} \mathbf{w}_g$$

$$\hat{\mathbf{p}}^f = \hat{\mathbf{p}}^l + \hat{\mathbf{p}}^g$$

Constitutive relations

$$\mathbf{T}^s = \mathbf{T}_E^s - n^s p\mathbf{I} = \lambda e\mathbf{I} + 2\mu\varepsilon_s - n^s p\mathbf{I}$$

with $e = \mathrm{div} u_s,\ \varepsilon_s = \mathrm{grad}^{sym} u_s$

$$p^{lR} = p_0^{lR} - K^l \varepsilon_l - \frac{K^l}{s_0^l}\left(s^l - s_0^l\right) - \frac{1-\phi_0}{\phi_0} K^l e \quad \left[\text{with } p^{lR} = p^{lR}\left(\rho^{lR}\right)\right]$$

barotropic fluid

$$p = s^l p^{lR} + s^g p^{gR}$$

$$p^c = p^{gR} - p^{lR}$$

Brooks and Corey model

$$p^c = p^b \left(s^{lR}\right)^{-\frac{1}{\lambda_{BC}}}$$

Porosity and saturation evolution

$$\phi = \phi_0 + \left(1-\phi_0\right)\mathrm{div} u_s$$

$$s^l = s_0^l + a_3\left[K^g \varepsilon_g - K^l \varepsilon_l - \frac{\left(1-\phi_0\right)\left(K^l - K^g\right)}{\phi_0} e\right]$$

Effective surface tension (spring)

$$\hat{\mathbf{p}}^f_{eq,2} = \xi\left(u_l - u_g\right)$$

Assumptions of continuum mixture

$$n^\alpha := \frac{dv^\alpha}{dv}, \ \sum_\alpha n^\alpha = 1, \ \forall \alpha \in \{s, l, g\}$$

$$\rho^{\alpha R} = \frac{dm^\alpha}{dv^\alpha} \text{ and } \rho^\alpha = \frac{dm^\alpha}{dv} \Rightarrow \rho^\alpha = n^\alpha \rho^{\alpha R}$$

$$\rho^{sR} = \text{const. (rigid grain assumption)}$$

$$\phi = n^l + n^g, \ s^l = \frac{n^l}{n^l + n^g} \text{ and } s^l + s^g = 1$$

$$a_3 = \left[\frac{s_0^g K^l + s_0^l K^g}{s_0^g s_0^l} + \frac{p^b}{\lambda_{BC}\left(1 - s^l_{res} - s^g_{res}\right)}\left(s_0^l\right)^{-\frac{1+\lambda_{BC}}{\lambda_{BC}}}\right]^{-1}$$

n^α = volume fraction
$\rho^{\alpha R}$ = effective density
ρ^α = partial density
ϕ = porosity
s = saturation
$\mathbf{u}$=displacement
$\mathbf{w}$=rel. fluid velocity
$\mathbf{T}$ = stress tensor
$\mathbf{I}$ = identity matrix
$\hat{\mathbf{p}}$=moment interaction
p=pressure
p^b=bubbling pressure
K=bulk modulus
γ=effective weight
k=effective permeability
λ=Lame parameter
μ=shear modulus
λ_{BC}=empirical parameter

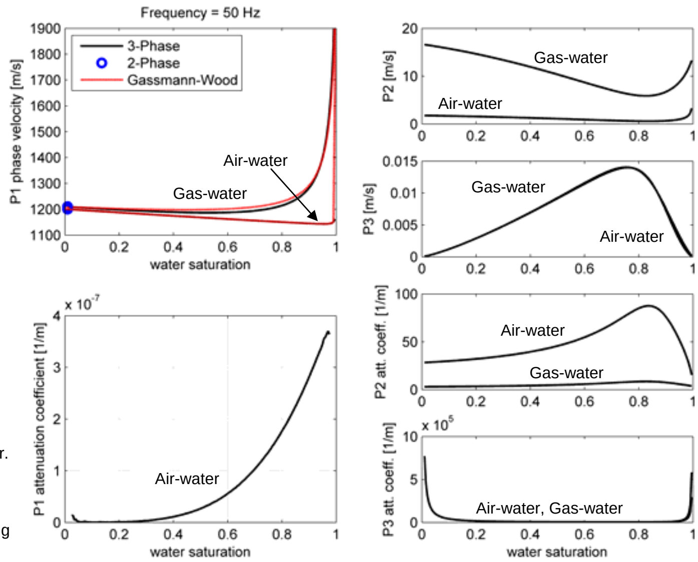
Results of the three-phase model (so far without surface tension)

We calculated the three P-wave phase velocities (P1, P2, P3) and the corresponding attenuation coefficients from the eigenvalues of the governing system of equations. The results of the Gassmann-Wood limit [e.g., 2] were calculated for a value of the bulk modulus of the solid grain of 35 GPa. The first P-wave (P1) phase velocity of the three-phase model assuming incompressible grains agrees well with the Gassmann-Wood limit assuming compressible grains. The results are calculated for a Sandstone partially saturated with either air and water or gas and water.

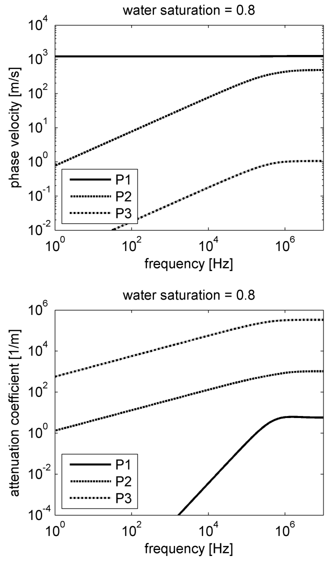
Material parameters of Massillon Sandstone (from [3], after [4])

Bulk modulus of matrix, GPa	1.02
Shear modulus, GPa	1.44
Density of solid grains, kg/m^3	2650
Intrinsic permeability, m^2	9 10^{-13}
Porosity	0.23
Bulk modulus of water, GPa	2.25
Density of water, kg/m^3	997
Viscosity of water, Pa s	1 10^{-3}
Bulk modulus of air, MPa	0.145
Density of air, kg/m^3	1.1
Viscosity of air, Pa s	1.8 10^{-5}
Bulk modulus of gas, GPa	0.022
Density of gas, kg/m^3	140
Viscosity of gas, Pa s	3 10^{-5}

The results for P1, P2, P3 and the attenuation coefficients agree well with the results of the three-phase model developed by [3] including compressibility of the solid grains. However, the coefficients in our model are considerably simpler. In our model the saturation is a dependent field variable, whereas it is constant e.g. in [5].
Our results agree well with the Gassmann-Wood limit and the results of [3] because the non-wetting phase is gas or air having a significantly smaller bulk modulus than the wetting phase and the solid grains.



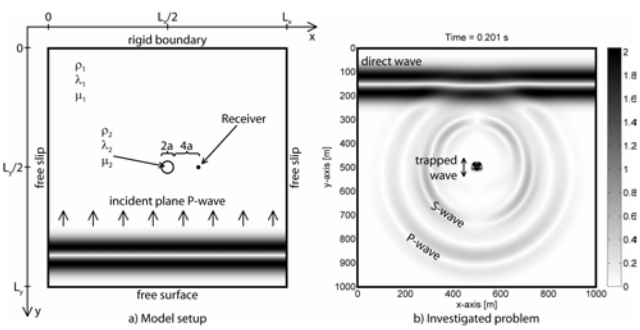
P-wave phase velocities and attenuation coefficients versus water saturation for both gas-water and air-water partial saturation. P1 agrees well with the Gassmann-Wood limit for both partial gas and air saturation.

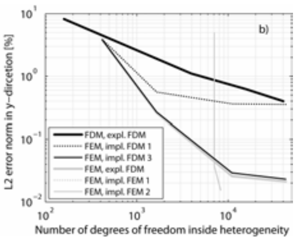


Dispersion relations of the three P-waves for gas-water saturation.

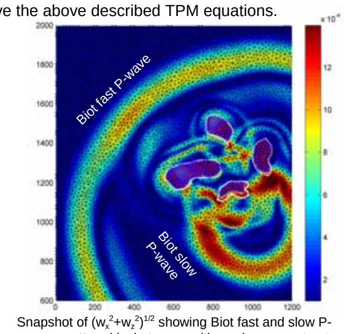
The way forward to numerical implementation

We compared finite difference and finite element methods for modeling two-dimensional (2D) scattering of elastic waves by a weak circular inclusion [11]. Time integration has been done with both explicit and implicit methods, also with an implicit finite element time integration. We compared the numerical results with an analytical solution for 2D scattering of elastic waves based on displacement potentials using Hankel functions [7]. The finite element method has been used to solve Biot's equation [9] and will be used to solve the above described TPM equations.
Main result: The finite element method with explicit time integration yields best results with respect to accuracy and computation time.





Results of accuracy tests for different numerical algorithms. The analytical solution is compared with the vertical displacement recorded at a synthetic receiver to the right of the circular inclusion (see left Figure a).



Snapshot of $(w_x^2+w_z^2)^{1/2}$ showing Biot fast and slow P-waves scattered by heterogeneities using an unstructured finite element mesh. The Biot equations [9] have been implemented using a velocity-stress formulation.

References: [1] R. de Boer.: Trends in Continuum mechanics of Porous Media, Springer, 2002. [2] Mavko et al., The Rock Physics Handbook, Cambridge University Press, 2003. [3] Tuncay & Corapcioglu, J. Geophys. Res., 1996. [4] Murphy, J. Acoust. Soc. Am., 1982. [5] Santos et al., J. Acoust. Soc. Am., 1990. [6] Brooks & Corey, Colorado State University, Hydrology Paper No. 3, 1964. [7] Liu et al., Geophys. J. Int., 2000. [8] Hilpert et al., Geophysics, 2000. [9] Biot, J. Appl. Phys., 1962. [10] Ehlers & Blum, Porous Media: Theory, Experiments and Numerical Applications, Springer, 2002. [11] Frehner et al., Phys. Earth Planet. Int., in press (online), 2008.