Theoretical and numerical modeling of waves in three-phase media

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We present theoretical and numerical models for wave propagation in three-phase media, i.e. one porous solid skeleton and two pore fluid phases. In this study we use the results of two different numerical models that employ particularly simple representations of a three-phase medium as a motivation to test and elaborate a theoretical three-phase model we derived recently. This phenomenological three-phase continuum model is based on the thermodynamical-consistent mixture theory extended by the concept of volume fractions. Commonly, it is denoted as the Theory of Porous Media.
We present two types of numerical wave propagation models: (1) a two-dimensional (2D) finite element model (FEM) for scattering in elastic, heterogeneous rocks, and (2) a three-dimensional (3D) finite difference model (FDM) of wave propagation in poroelastic rocks. In the 2D FEM, scattering of a plane wave at a circular inclusion with smaller elastic bulk modulus than the surrounding material is simulated (Fig. 1). The FEM employs an unstructured, triangular mesh to resolve the circular inclusion accurately. The results of the FEM are compared with an analytical solution for the investigated scattering problem and agree well. The Fourier spectrum of the initially plane wave is modified by the scattering and characteristic anomalies are visible which occur at the eigenfrequencies of the circular inclusion.

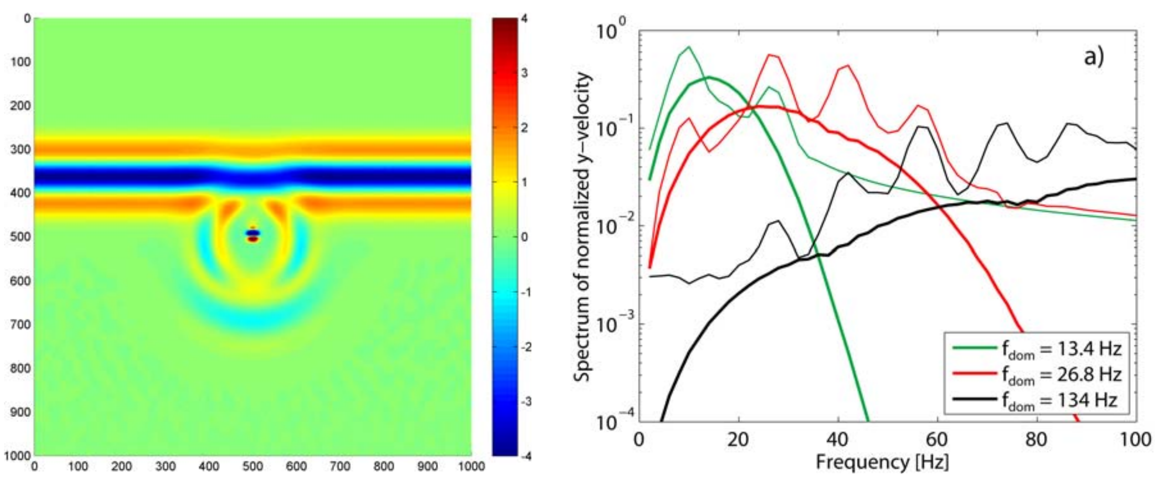

Figure 1. Left: Snapshot of 2D finite element model showing scattered wave field after a plane wave passed the circular inclusion in the middle of the model domain (arbitrary units). Spectra of three different initial plane waves with different central frequencies (thick lines) and the corresponding spectra of the scattered wave fields after the plane wave passed the circular inclusion. Characteristic anomalies are visible.

The 3D finite difference model resolves the 3D pore geometry accurately and models the solid skeleton as an elastic material whereas it models the pore fluids as viscous materials (Fig. 2). Therefore, local fluid flow effects and scattering on the pore scale are included in the model. We apply model parameters that describe the mechanical behavior of sandstone where pores are either filled with water or are dry. The porous medium is bounded on two sides with a purely elastic medium to emit initially a well defined plane wave.

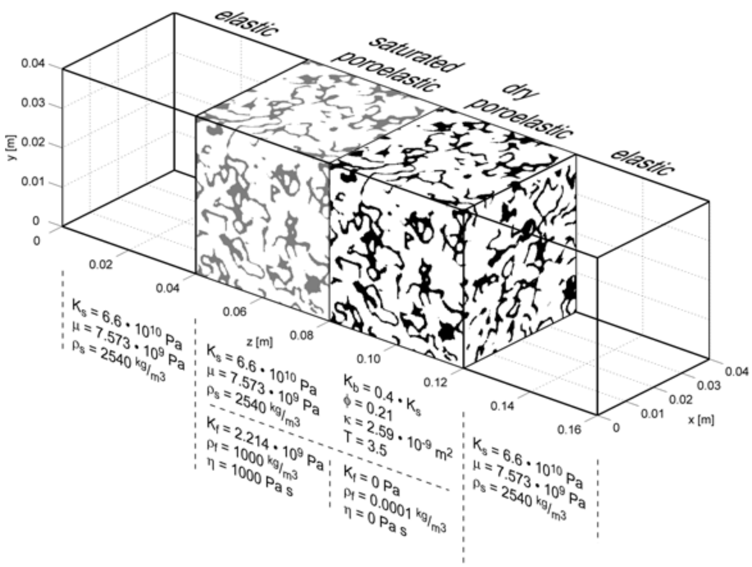

Figure 2. Model set-up and material parameters used for the 3D finite difference model. The elastic porous skeleton and the viscous pore fluid are fully resolved numerically so that local flow and scattering on the pore scale can occur.

The two different numerical models can be considered as the simplest representation of a partially-saturated rock in the low-frequency range neglecting capillary forces. The homogenized results obtained by these approaches in a statistically Representative Volume Element (RVE) are identical to a macroscopic continuum-based three-phase model. Moreover, the three-phase model allows studying a heterogeneous distribution of the field of saturation in a geophysical-relevant domain much larger then the RVE under study. Furthermore, the variation of degree of saturation in space affects the state of pore pressure through the capillary-pressure relation and therefore the effective phase velocity and attenuation of the overall three-phase mixture.

Our continuum three-phase model predicts three compressional waves in each phase and one shear wave in the solid. Capillary pressure depending on the degree of saturation is taken into account by the empirical van Genuchten equation. Effects of capillary pressure are not taken into account in the two numerical models. We will discuss the transition from the microscale numerical models, including scattering and local fluid flow, to the macroscale continuum models without scattering and local fluid flow. Also we will discuss the impact of the capillary pressure on the wave propagation and attenuation by comparing our three-phase model with two-phase models (e.g. Biot model) that mimic partial saturation by varying material properties in space.

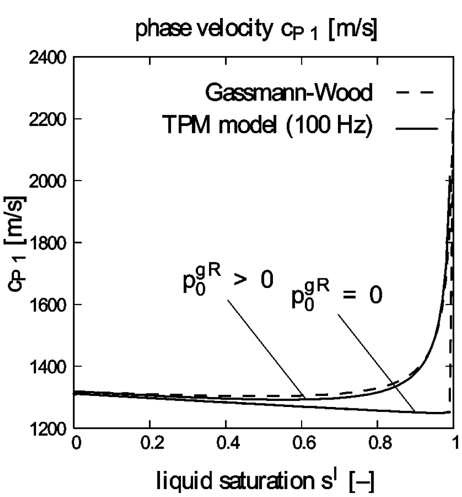

Figure 3. Results of the three-phase model. The first compressional wave velocity in the solid (Cp1) is plotted versus the saturation of the liquid phase (the third phase is considered to be gas). The results of the three-phase model agree well with the prediction of the Gassmann-Wood model that is valid in the low-frequency range. Results are shown for two different values of the effective gas pressure (p_0^{gr}) corresponding to different burial depths of the corresponding rocks.