Fold growth rates in 3D buckle folds

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Abstract

Geological folds are inherently 3D structures; therefore, they also grow in three dimensions. Here (Frehner, 2014), fold growth in all three dimensions is quantified by numerically simulating upright single-layer folds in 3D Newtonian media. Horizontal uniaxial shortening leads to a buckling instability, which grows from a point-like initial perturbation in all three dimensions by fold amplification (vertical), fold elongation (parallel to fold axis) and sequential fold growth (parallel to shortening direction) of secondary (and further) folds adjacent to the initial isolated fold. The two lateral directions exhibit similar averaged growth rates, leading to bulk fold structures with aspect ratios in map view close to 1. However, fold elongation is continuous with increasing bulk shortening, while sequential fold growth exhibits jumps whenever a new sequential fold appears and the bulk fold structure therefore suddenly occupies more space. Compared with the two lateral growth directions, fold amplification exhibits a slightly higher growth rate.

Definitions of 3D Fold Growth

Fold amplification: (z-direction)	Growth from a fold shape with low limb-dip angle to a shape with larger limb-dip angle.
Fold elongation: (y-direction)	Parallel to fold axis. Growth from a dome-shaped (3D) structure to a more cylindrical fold (2D).
Sequential fold growth: (x-direction)	Parallel to shortening direction. Growth of additional folds adjacent to initial fold. The initial fold is termed 0th sequential fold; later folds are numbered consecutively.

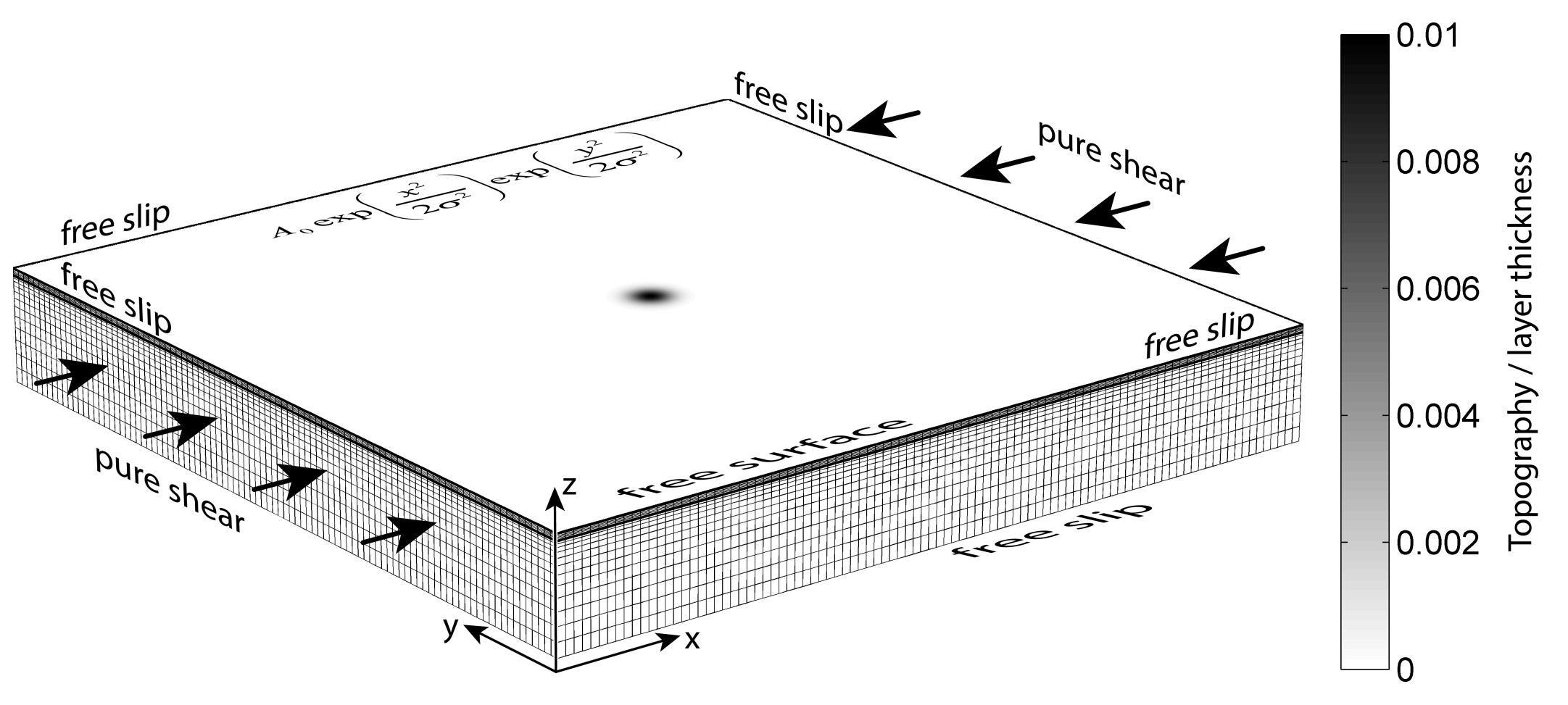


Fig. 1: Initial numerical grid, boundary conditions, and coordinate system for studying 3D fold growth. A higher-viscosity layer is resting on a lower-viscosity layer. Grey values represent the initial topography (equation in the figure). Arrows indicate the pure-shear shortening boundary condition in the x-direction.

Model Setup and Numerics

- Rheology: Incompressible Newtonian
- Two layers (Fig. 1) with viscosity ratio R
- 2D Gaussian initial perturbation (Fig. 1) with $A_0/h_{upper}=0.01$

Numerical method

- 3D finite-element (FE) method using Lagrangian numerical grid
- Mixed velocity-pressure-penalty formulation (Galerkin method)
- Isoparametric cubic Q27/4 elements
- Uzawa-type iteration to enforce incompressibility

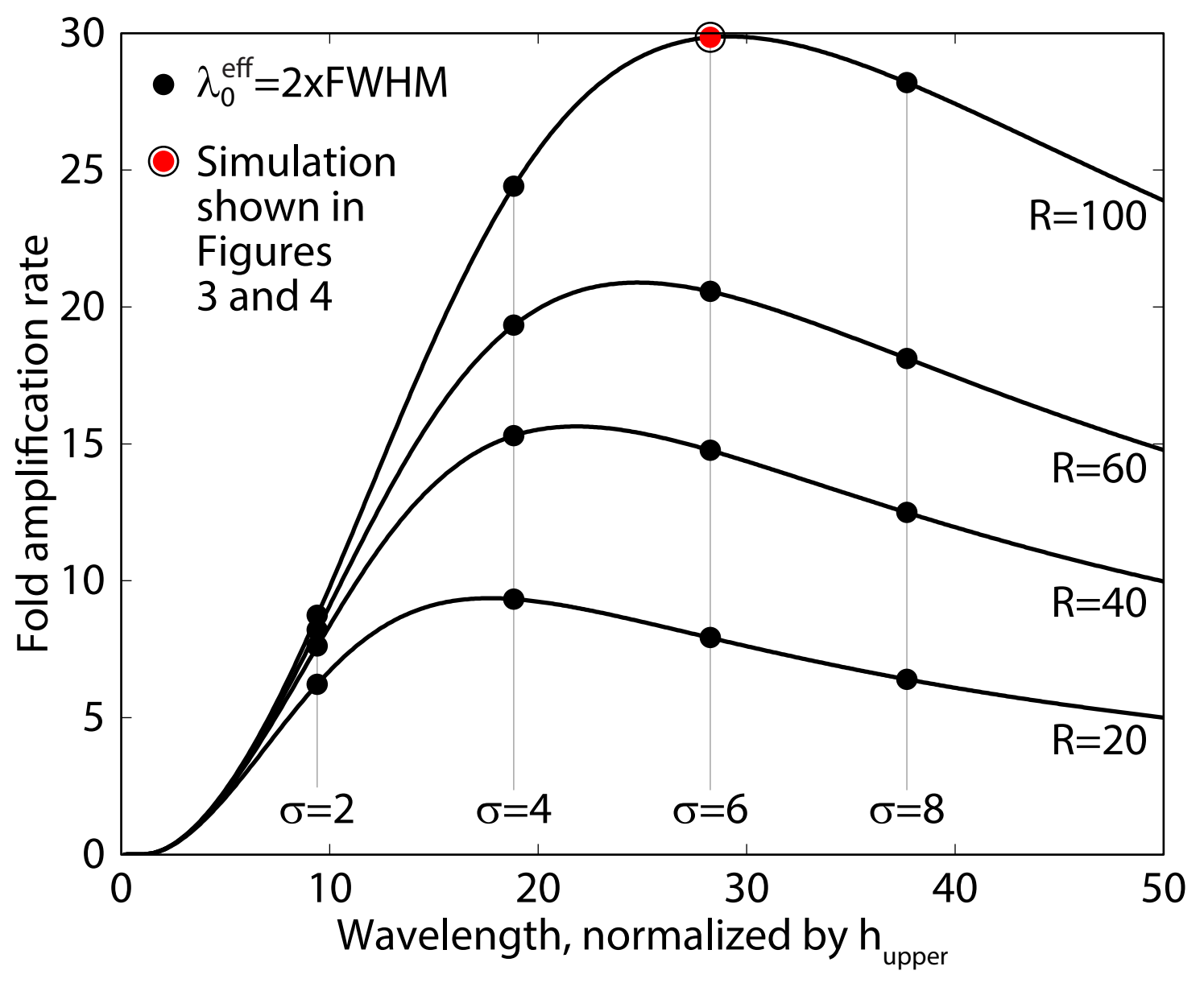


Fig. 2: Analytical fold amplification-rate spectra for different viscosity ratios, R (Fletcher, 1991). Dots indicate σ-values used in the numerical simulations. FWHM: Full width at half maximum.

Boundary conditions (Fig. 1)

- Free surface on top surface
- Free-slip boundaries perpendicular to y-coordinate and on bottom boundary
- Free-slip moving wall conditions on boundaries perpendicular to x-coordinate: x-velocity is modified each time step to enforce constant pure-shear strain rate

Simulation Snapshots

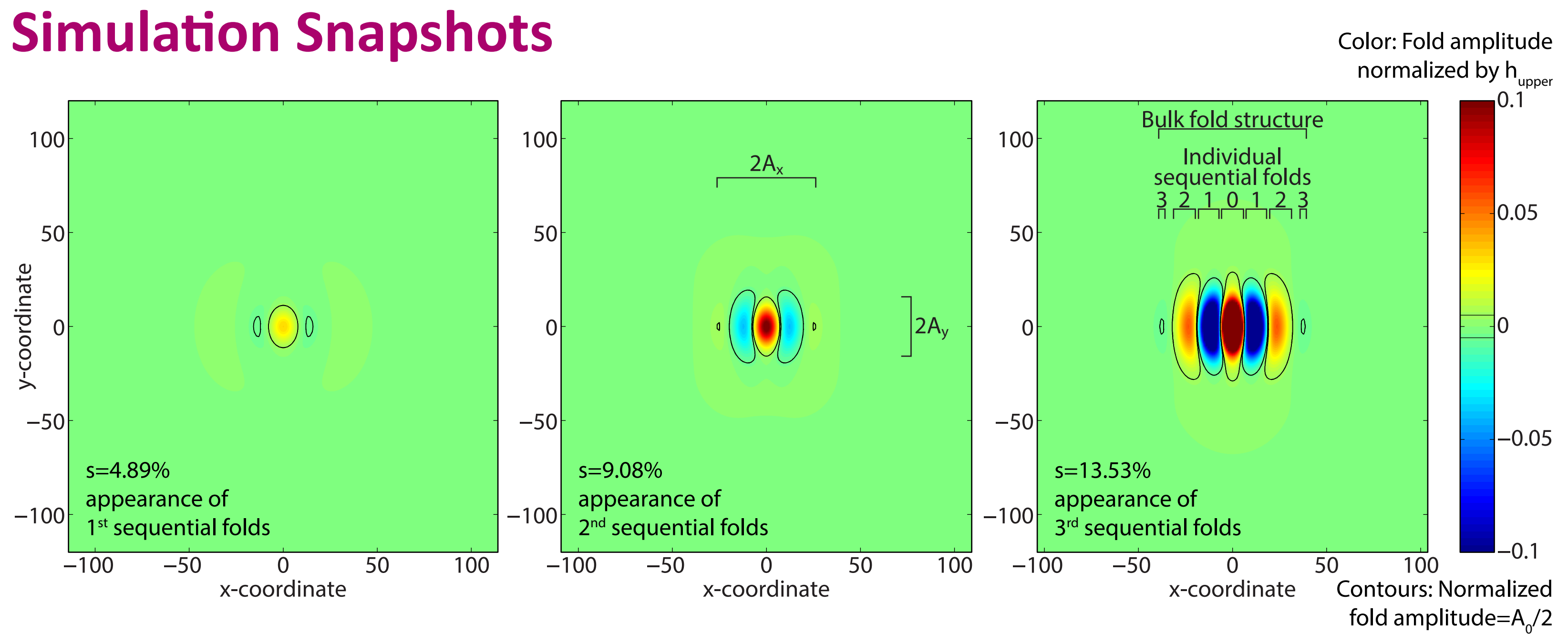


Fig. 3: Simulation snapshots of a typical FE-simulation showing the evolution of the 3D fold structure in top-view with increasing shortening, s.

Calculations based on FE-simulations

From the numerical simulations the fold amplitudes in all thre directions are (Fig. 3):

Amplitude in z-direction: $A_z = z\big|_{x=0,y=0} - z_{\mathrm{ref}}$ } Corresponds to central color in Fig. 3

Amplitude in y-direction: $A_y = \max(y) \text{ where } z\big|_{x=0} - z_{\mathrm{ref}} = \frac{A_0}{2}$

Amplitude in x-direction: $A_x = \max(x) \text{ where } z\big|_{y=0} - z_{\mathrm{ref}} = \frac{A_0}{2}$

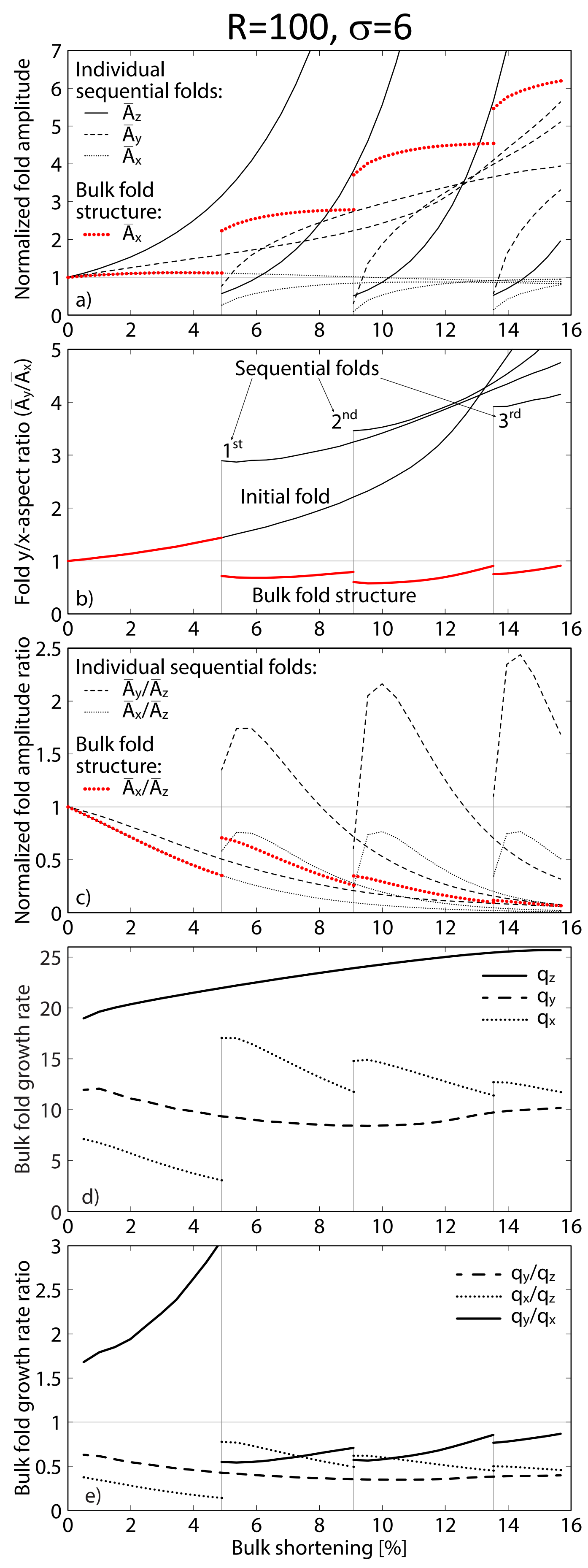
} Corresponds to black contour lines in Fig. 3

z_{ref} is the average topography of the upper model surface. Assuming exponential growth of the fold structure in 3D, growth rates can be calculated as (t is time):

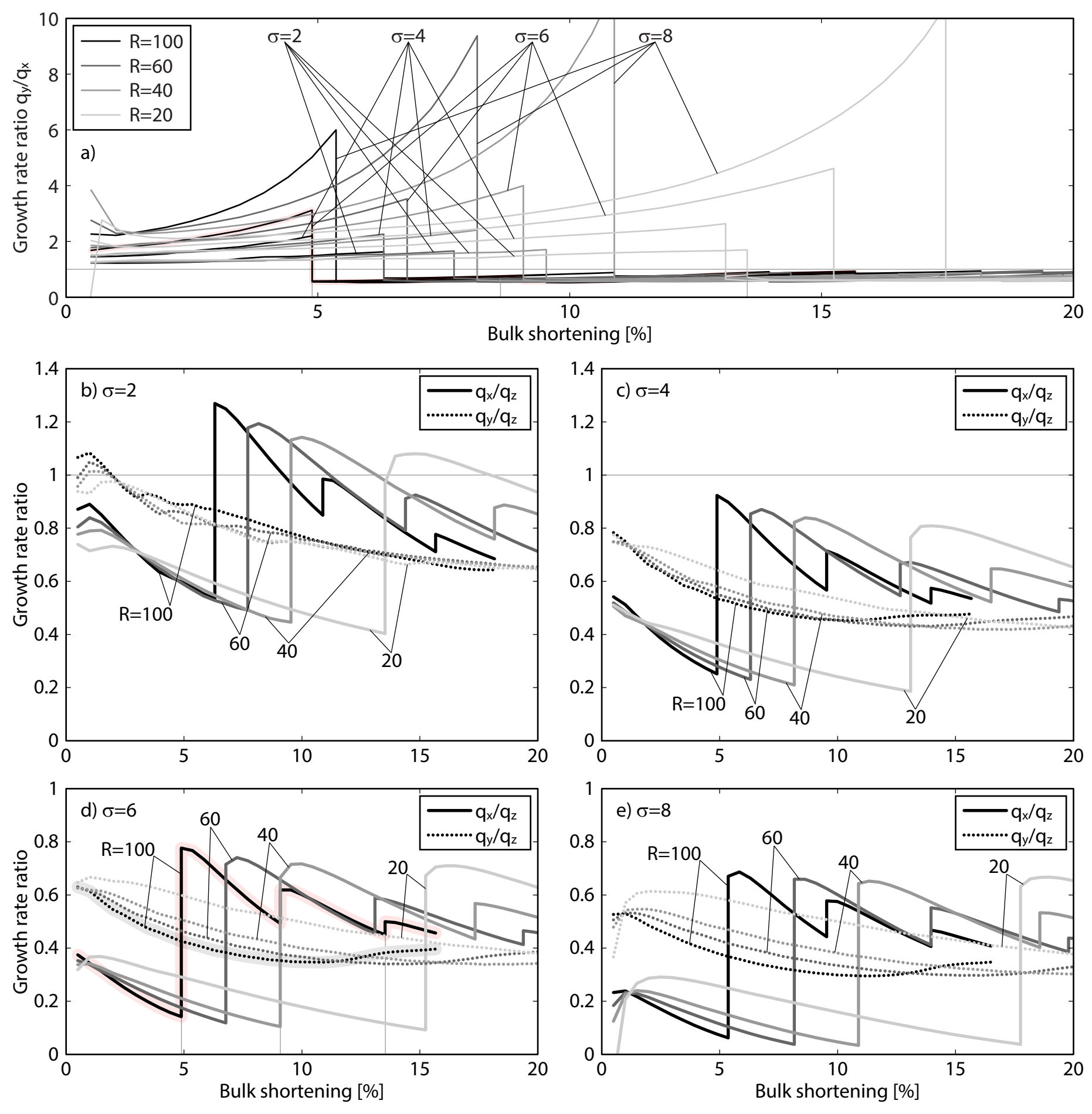
$$q_i = -\frac{1}{\dot{\varepsilon}_{xx} t} \ln\left[\frac{A_i}{A_i\big|_{t=0}}\right] + \begin{cases} -1 & \text{for } i = z \\ 0 & \text{for } i = y \\ 1 & \text{for } i = x \end{cases}$$

The different summands originate from the background deformation field, which kinematically amplifies the initial perturbation differently in the different directions.

3D Fold Amplitudes and Growth Rates



←Fig. 4: Fold amplitude evolution in all three directions (a), fold y/x-aspect ratio (b), fold amplitude ratio (c), fold growth rates (d), and ratios between fold growth rates (e) for both the individual sequential folds and the bulk fold structure (thick lines) with increasing shortening for the same simulation as shown in Fig. 3. All amplitudes are normalized by their initial value. b) and c) are derived from a); d) is derived from a) by applying the equations above; e) is derived from d). Vertical gray lines indicate first appearance of sequential folds.



↑Fig. 5: Same as Fig. 4e) but for all other viscosity ratios, R, and initial conditions, σ, and plotted in two subfigures. The curves shown in Fig. 4 are decorated with light red background.

Discussion and Conclusions

- Growth modes in the three directions are different: Amplification and elongation are primarily related to the initial isolated fold; sequential growth is due to the appearance of new folds. Additionally, the two lateral growths involve more and more material; amplification does not incorporate significantly more material.
- Growth in x-direction of the individual folds (Fig. 4a) is limited to an amplitude of around 1 showing that the bulk fold structure grows in x-direction by sequential folding and not by the growth of one individual syn- or antiform.
- Sequential growth of the bulk fold structure (Fig. 4a) is marked by sudden jumps every time a new sequential fold appears. However, the average x-amplitude (and growth rate) is similar as in y-direction leading to a nearly constant bulk y/x-aspect ratio (and growth rate ratio) of ~1 (Fig. 4b and 5a).
- Amplitude ratios (and growth rate ratios) with the z-direction as denominator (Fig. 4c and 5b-e) decrease with increasing shortening, indicating that fold growth in the z-direction exhibits a higher rate than those in the two lateral directions.

Reference: Frehner M.: 3D fold growth rates, in press in Terra Nova, doi:10.1111/ter.12116. **Acknowledgements:** Neil Mancktelow and Naiara Fernandez