3D fold growth rates

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INTRODUCTION

Geological folds are inherently three-dimensional structures. Therefore, a fold structure also grows in three dimensions. In this study, fold growth in all three dimensions is studied and quantified numerically using a finite-element algorithm for simulating three-dimensional deformation of Newtonian materials. The horizontal higher-viscous layer exhibits an initial point perturbation. Horizontal compression in one direction (x-direction) leads to a mechanical folding instability, which grows from this perturbation in all three dimensions.

DEFINITIONS

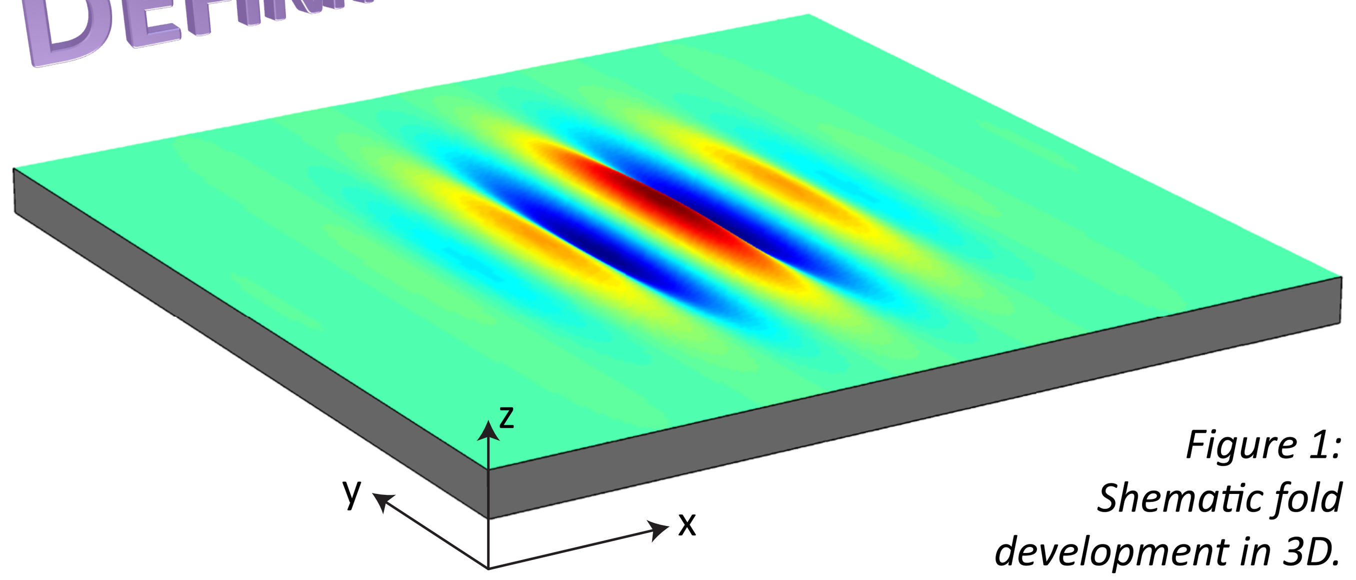


Figure 1: Shematic fold development in 3D.

Fold amplification (growth in z-direction)

Fold amplification (vertical) describes the growth from a fold shape with low limb-dip angle to a shape with higher limb-dip angle.

Fold elongation (growth in y-direction)

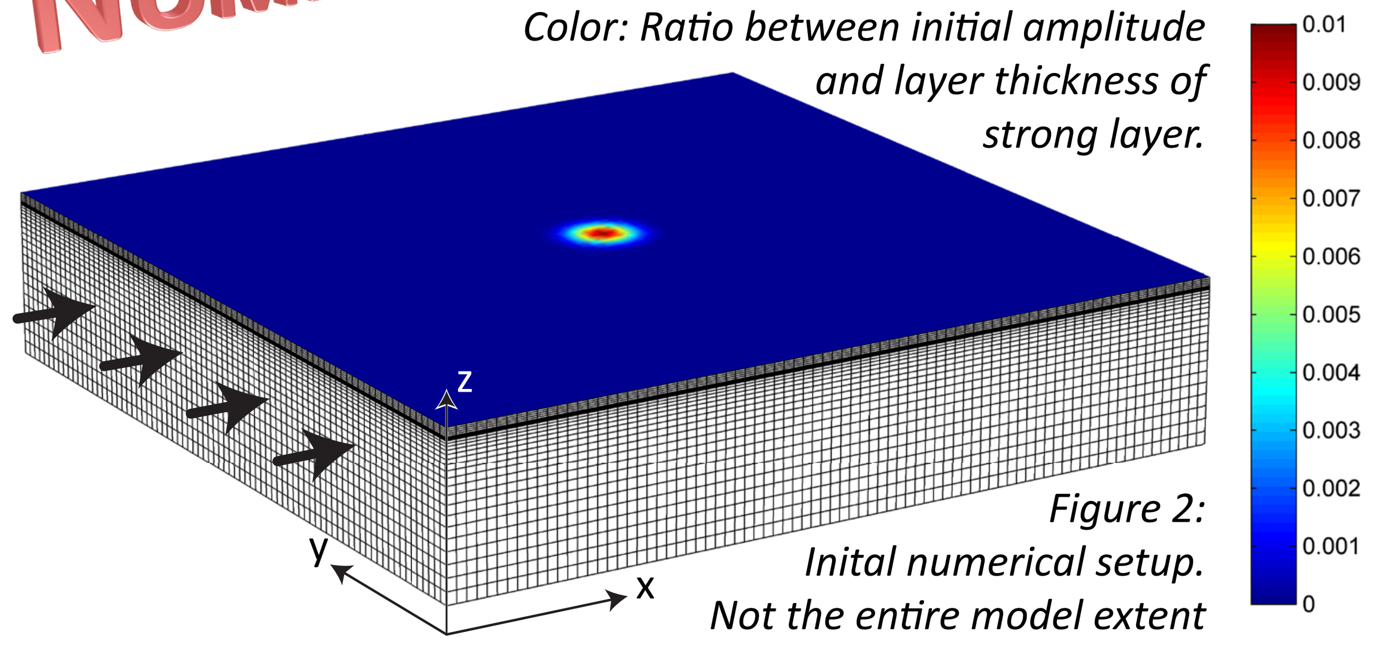
Fold elongation is parallel to the fold axis and describes the growth from a dome-shaped (3D) structure to a more cylindrical fold (2D).

Sequential fold growth (growth in x-direction)

Sequential fold growth is parallel to the shortening direction and describes the growth of secondary (and further) folds adjacent to the initial isolated fold.

Both fold elongation and sequential fold growth have been referred to as **lateral fold growth**, which is here used as an umbrella term for both.

NUMERICS



Color: Ratio between initial amplitude and layer thickness of strong layer.

Figure 2: Inital numerical setup. Not the entire model extent in x- and y-direction is shown.

Numerical method

- 3D finite-element method using Lagrangian numerical grid
- Mixed velocity-pressure-penalty formulation (Galerkin method)
- Isoparametric cubic Q27/4 elements (Figure 3);
- Shape functions: tri-quadratic continuous for velocity; linear discontinuous for pressure
- Uzawa-type iteration to enforce incompressibility

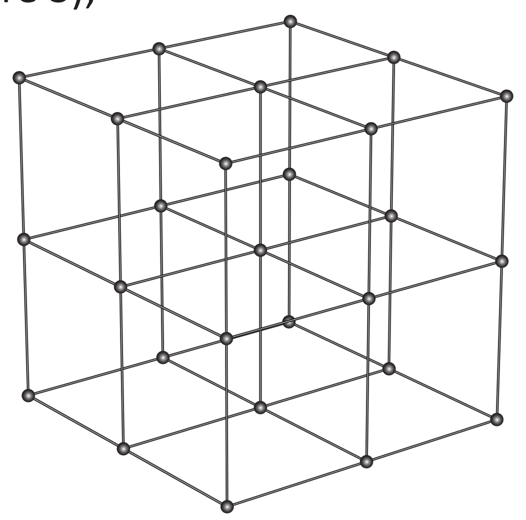


Figure 3: Isoparametric cubic Q27/4 element.

Model setup and initial conditions

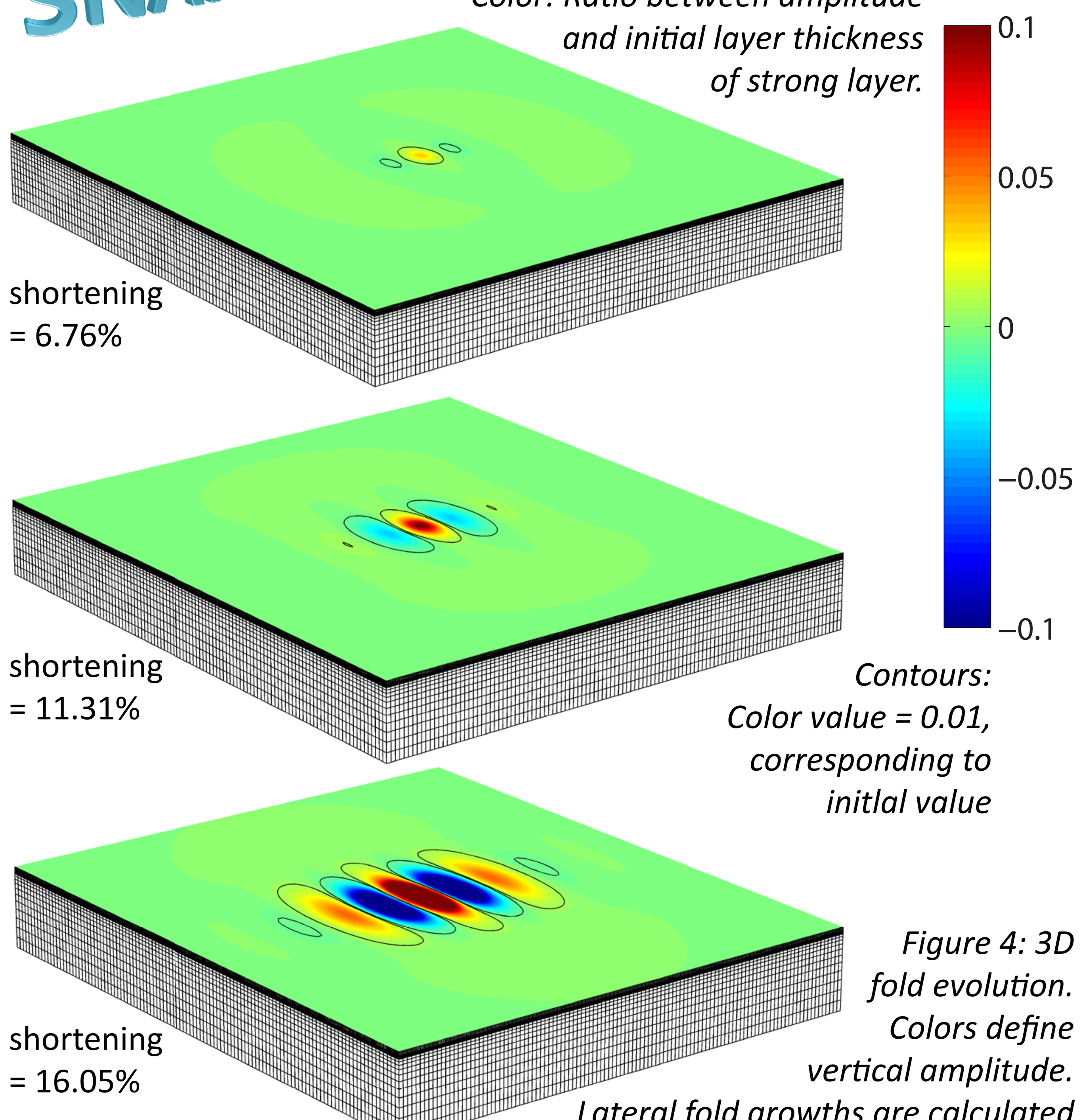
- Rheology: Incompressible Newtonian
- Two layers: low-viscosity lower layer, high-viscosity upper layer
- Viscosity ratio: $R=\eta_{upper}/\eta_{lower}=100$
- Thickness ratio: $h_{lower}/h_{upper}=15$
- Perfectly welded interface between the two layers
- 2D Gaussian (G) with a distance between inflexion points, $\sigma/h_{upper}=4$ and maximal amplitude, $A_0/h_{upper}=0.01$ used as initial perturbation on both interfaces of the upper layer (Figure 2)

$$G = A_0 \exp\left(\frac{x^2}{2\sigma^2}\right)\exp\left(\frac{y^2}{2\sigma^2}\right)$$

Boundary conditions

- Free-slip (zero traction) non-moving walls perpendicular to y-coordinate and on basal boundary perpendicular to z-coordinate
- Free surface (zero traction) on top surface
- Free-slip (zero traction) moving walls perpendicular to x-coordinate: x-velocity is applied to enforce pure-shear deformation.

SNAPSHOTS



Color: Ratio between amplitude and initial layer thickness of strong layer.

Contours: Color value = 0.01, corresponding to initlal value

Figure 4: 3D fold evolution. Colors define vertical amplitude. Lateral fold growths are calculated from horizontal extent of contour lines.

CALCULATION

First, the amplitudes in the three directions are caluclated as follows:

- **Amplitude in z-direction** (Fold amplification) $A_z = z|_{x=0,y=0} - z_r$
- **Amplitude in y-direction** (Fold elongation) $A_y = \max(y)$ where $\left(z|_{x=0} - z_r\right) > A_0/2$
- **Amplitude in x-direction** (Sequential fold growth) $A_x = \max(x)$ where $\left(z|_{y=0} - z_r\right) > A_0/2$

Two different reference z-values, z_r, are used:

- z_{mean}: mean z-value of one interface (solid lines in Figures 5 & 6, also used in Figure 4)
- z_{corner}: corner z-value of one interface (dotted lines in Figures 5 & 6)

The growth rates are calculated assuming exponential growth:

- **Growth rate in z-direction** $q_z = -\ln\left(A_z/A_0\right)/\dot{\varepsilon}_{xx}t - 1$
- **Growth rate in y-direction** $q_y = -\ln\left(A_y/\left(\sqrt{2\ln 2}\sigma\right)\right)/\dot{\varepsilon}_{xx}t$
- **Growth rate in x-direction** $q_x = -\ln\left(A_x/\left(\sqrt{2\ln 2}\sigma\right)\right)/\dot{\varepsilon}_{xx}t + 1$

3D GROWTH RATES

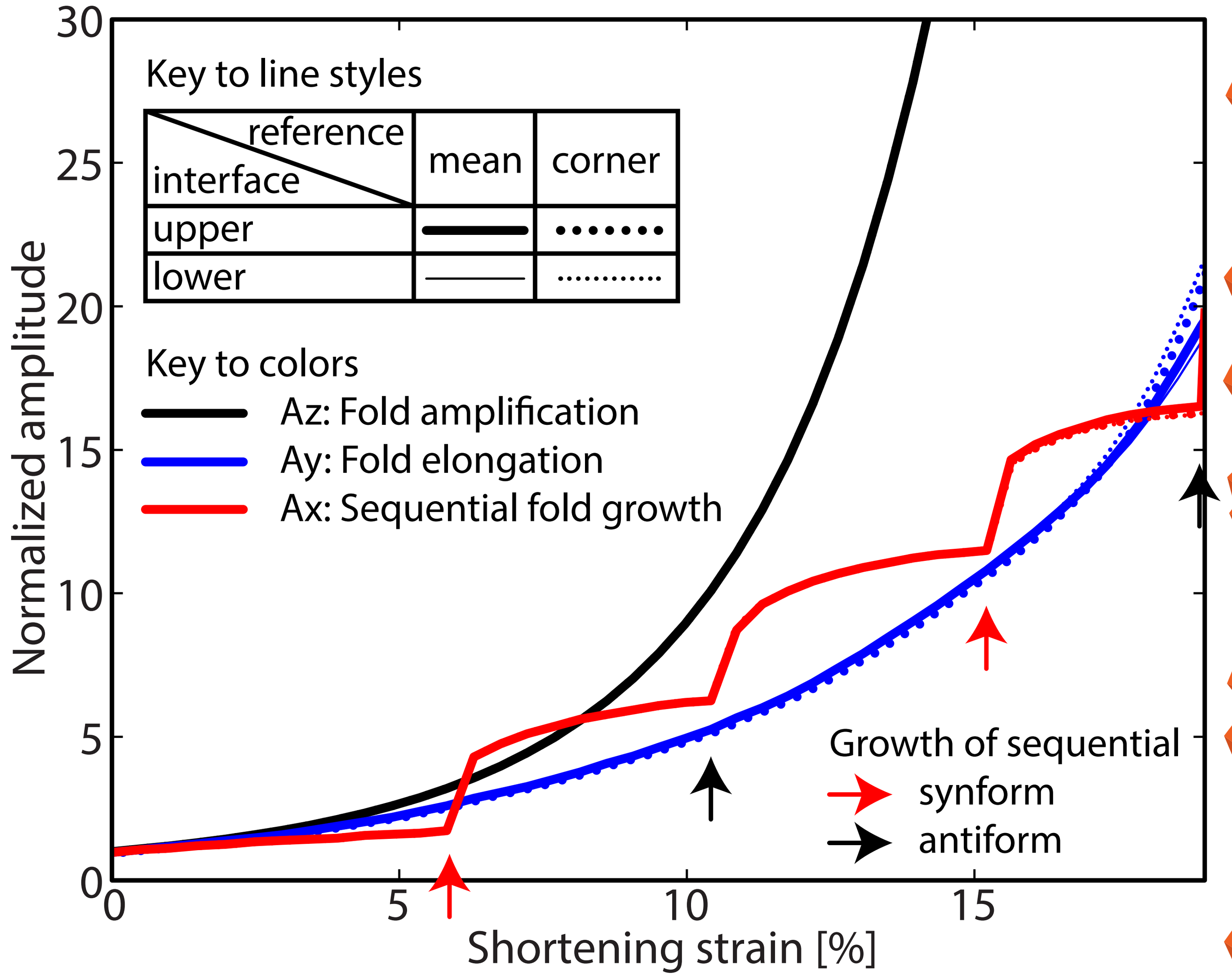


Figure 5: Progressive 3D amplitude evolution. Normalizations: A_0 for A_z, $\sqrt{2\ln(2)}\sigma$ for A_y and A_x.

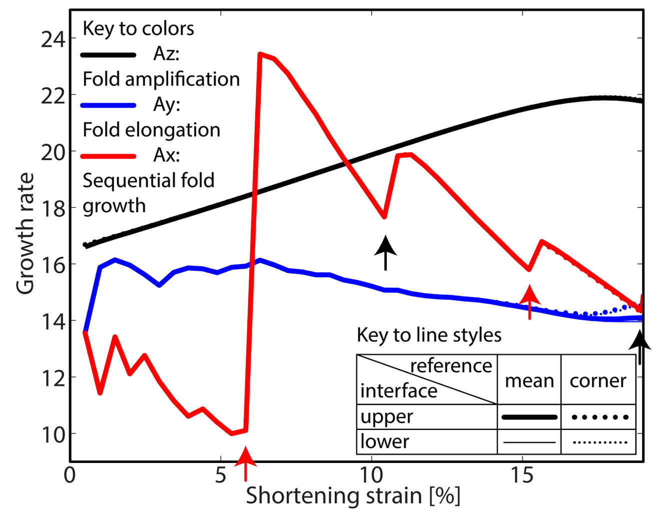


Figure 6: Fold growth rates as a function of bulk shortening in x-direction.

DISCUSSION & CONCLUSIONS

"Jumps" in sequential fold growth (Figure 5 & 6) occur when a growing adjacent fold reaches a vertical amplitude of >A_0/2. The average sequential fold growth may be approximated by an exponential function, as the other growths.

The following conclusions can be drawn:

- The fold growth rate (z-direction) is larger than the two lateral fold growth rates (x- and y-direction), particularly at later folding stages.
- The sequential fold growth rate (x-direction) is initially smaller than the fold elongation rate, increases after the first synform appears, and then decreases towards the value of the fold elongation rate.

These results may be applied to fold-and-thrust belts elongating along strike and advancing towards the foreland, e.g. the Zagros mountain belt.