Boundary effects in physical models of simple shear

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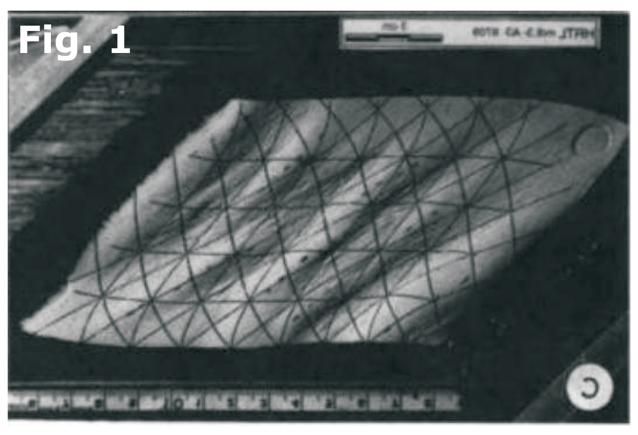
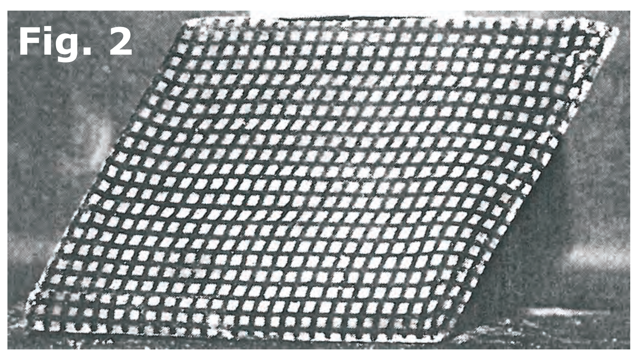
[1] SUMMARY

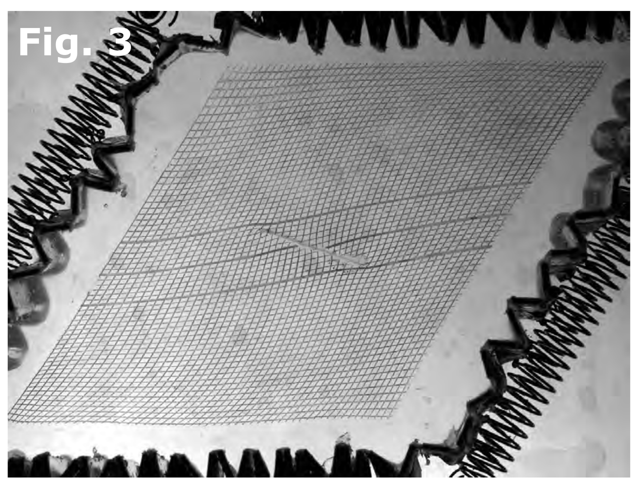
Analogue modeling of geological structures, such as the behavior of inclusions in a matrix or folding instabilities commonly employs a linear simple shear or general shear rig. In theory, a homogeneous plane strain flow is prescribed at the boundaries of such deformation rigs, but, in practice, the resulting internal deformation of the analogue material (commonly paraffin wax or silicone putties) often strongly deviates from the intended homogeneous strain field. This can easily lead to misinterpretation of such analogue experiments.

We present a numerical finite element study to quantify the influence of imperfect simple shear boundary conditions on the internal deformation of a homogeneous viscous analogue material. The results demonstrate that imperfect circumferential boundary conditions in the simple shear plane (*x*-*y*-plane) lead to the heterogeneous strain observed in some analogue experiments, depending on their design. However, in other experiments, the analogue material lies on top of a weak lubricating material (e.g., Vaseline) or is sandwiched between two such materials. These layers lead to a viscous drag force acting on the surface of the analogue material that represents imperfect simple shear boundary conditions in the third dimension (*z*-direction). For this experimental configuration, the numerical results show that the lubricating layers are responsible for the heterogeneous strain observed in analogue models. The resulting errors in internal strain can be as high as 100%. These important boundary effects, which are difficult to avoid, must be considered when interpreting analogue simple shear experiments.

[2] OBSERVED BOUNDARY EFFECTS IN ANALOGUE MODELS





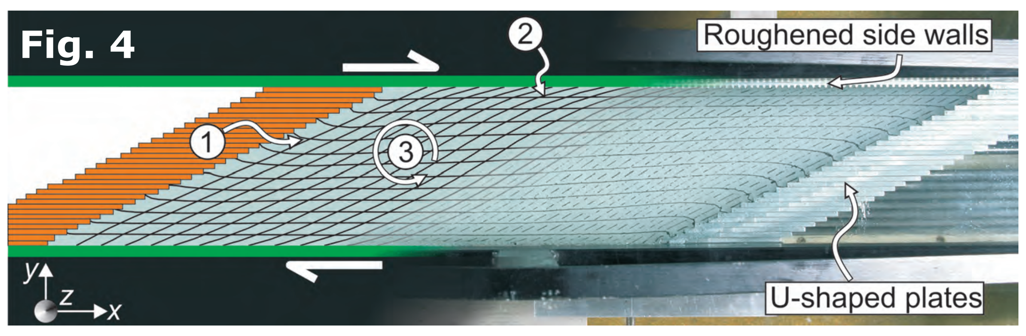


Analogue simple shear experiments of homogeneous media performed in different labs:

- Fig. 1: Hans Ramberg Tectonic Lab, Uppsala, Sweden (from Sengupta & Koyi, 2001)
- Fig. 2: CSIRO, Canberra, Australia (from Price & Torok, 1989)
- Fig. 3: Tectonophyscis Lab, Mainz, Germany (from Exner, 2005)
- Fig. 4: ETH Zurich, Switzerland (unpublished)

In Fig. 1 and 2, initially vertical passive lines tend to have a large angle with the boundaries.

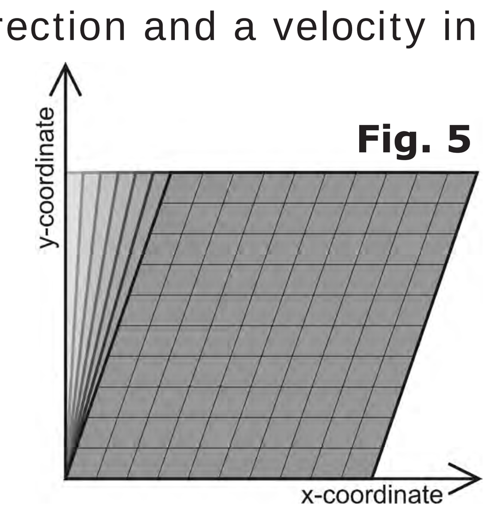
In Fig. 3 and 4, initially vertical passive lines tend to have a small angle with the boundaries.



[3] WHAT ARE SIMPLE SHEAR BOUNDARY CONDITIONS?

For perfect simple shear, at each boundary two boundary conditions need to be applied, i.e., zero velocity in the *y*-direction and a velocity in the *x*-direction depending on the position along *y*.

Condition / Boundary	$v_x = \dot{\gamma}_{ext} y$	$v_y = 0$
y-boundaries	✓	✓
x-boundaries	✓	✓



DEFINITION

x-Boundary: Initially perpendicular to x-axis

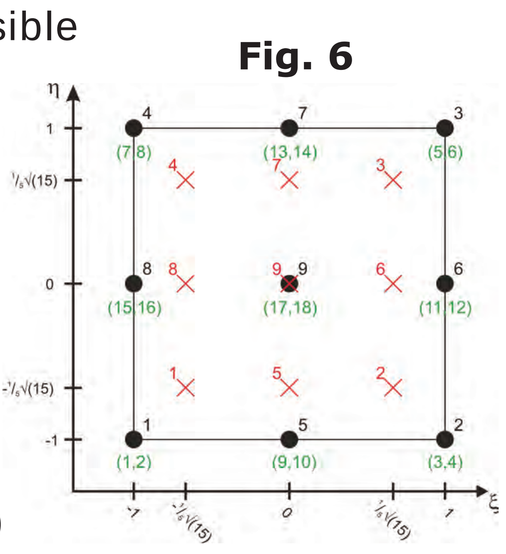
y-Boundary: Perpendicular to y-axis

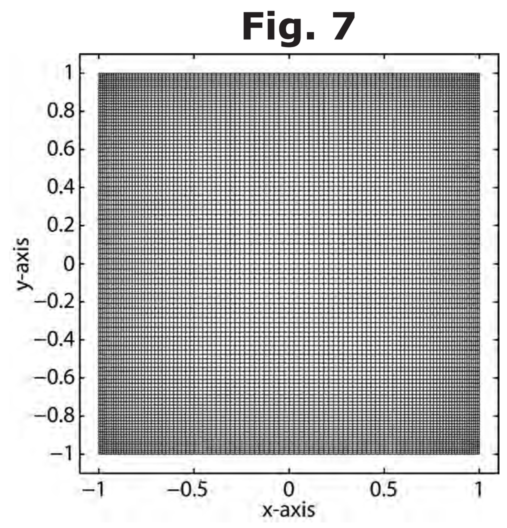
[4] NUMERICAL METHOD

Rheology:
Slow deformation of incompressible Newtonian viscous fluid

Finite element method:
- 9-node element (Fig. 6)
- Mixed v-p-formulation
- Penalty formulation
- Uzawa iteration
- Deforming Lagrangian grid

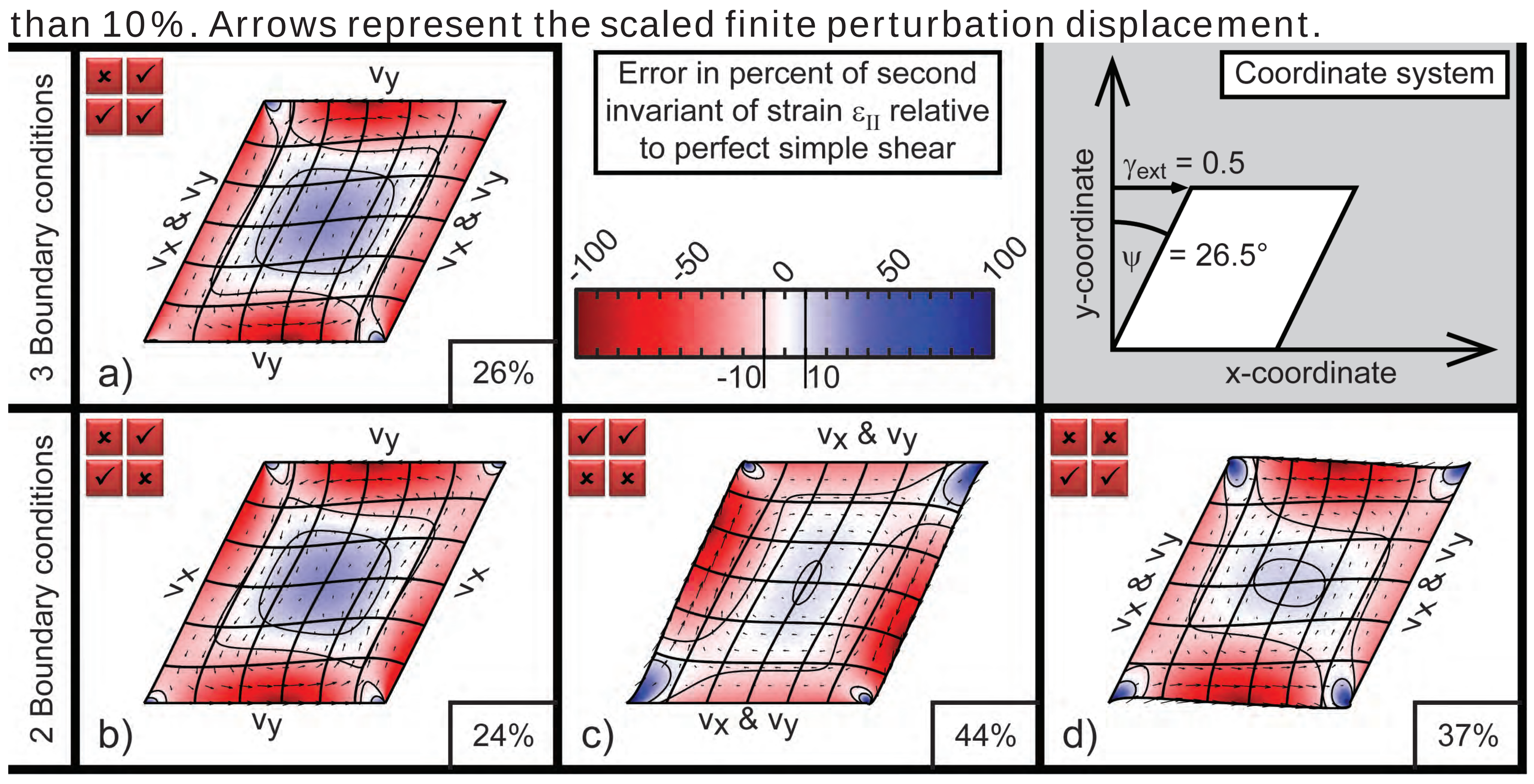
Resolution:
10'609 numerical nodes (Fig. 7)





[5] IMPERFECT X-Y-BOUNDARY CONDITIONS

Fig. 8: Numerically deformed homogeneous square in simple shear with an applied shear strain γ_{ext}=0.5. For perfect simple shear (experiment in upper right corner), four boundary conditions need to be applied (see Table). In a), only three and in b) to d), only two boundary conditions are applied (indicated in the same way as in the table and noted at each boundary). Thick black lines are passive marker lines. The color represents the second invariant of finite strain, ε_{II}, plotted as the error in percent relative to perfect simple shear. Thin black lines are the ±10% contour lines. Percentage values are the area of the model with an absolute error smaller than 10%. Arrows represent the scaled finite perturbation displacement.



[6] IMPERFECT Z-BOUNDARY CONDITIONS

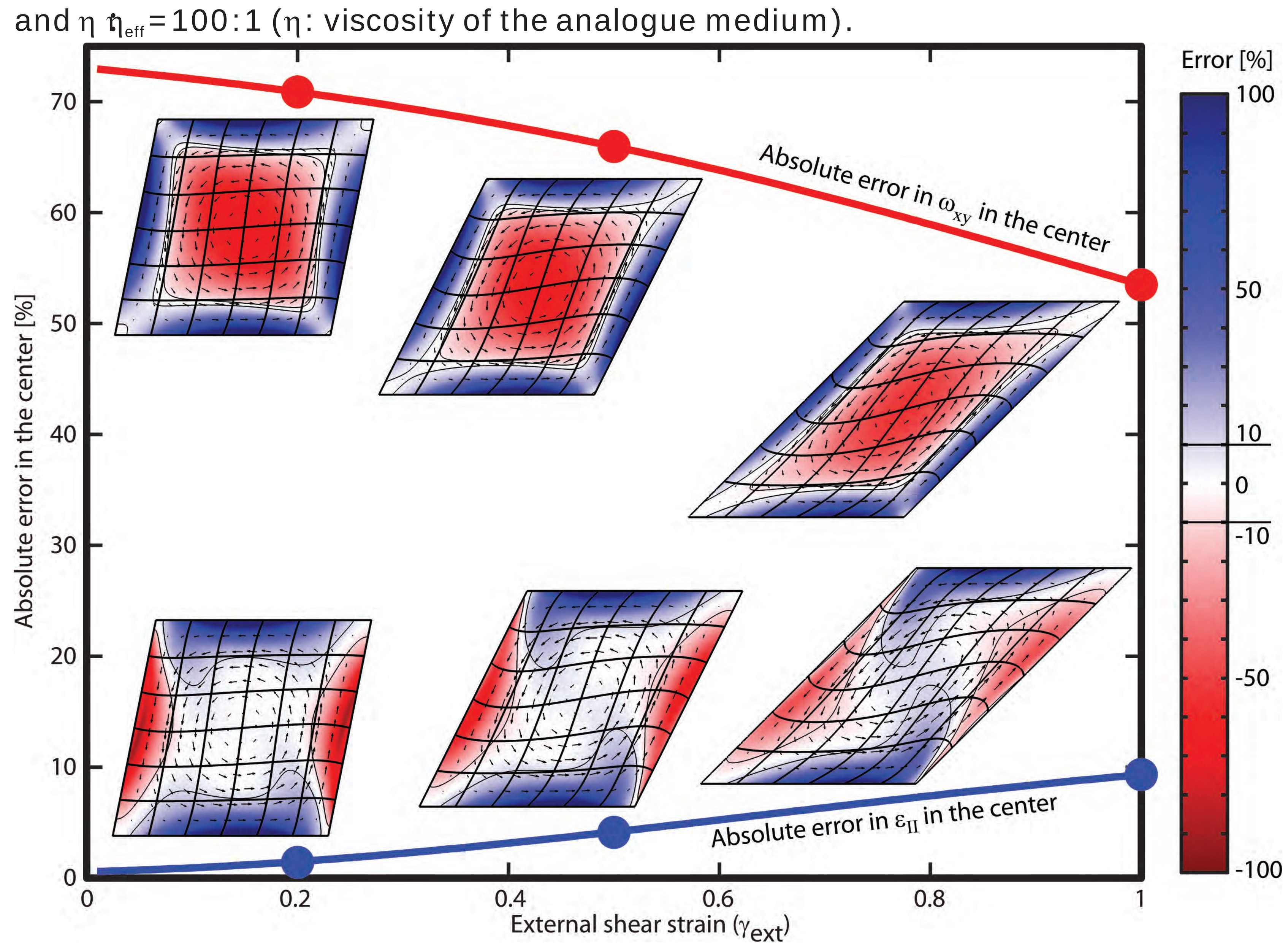
Lubricating layer in the z-direction corresponds to a velocity-dependent viscous drag force:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \frac{2\eta_{eff}}{hL_y} v_x$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \frac{2\eta_{eff}}{hL_y} v_y$$

σ_{ij} Stress tensor components
η_{eff} Effective viscosity lubricating layer
h Height of lubricating layer
L_y Model size in y-direction
v_i Velocity components

Fig 8: Numerically deformed homogeneous square in simple shear with increasing applied shear strain γ_{ext} and perfect simple shear boundary conditions in the x-y-plane, but viscous drag boundary conditions in the z-direction. In the inset figures, the color represents the second invariant of finite strain, ε_{II}, (lower inset figures) and the finite spin, ω_{xy}, (upper inset figures), respectively, both plotted as the error in percent relative to perfect simple shear. Thin black lines are the ±10% contour lines. Thick black lines are passive marker lines. Arrows represent the scaled finite perturbation displacement. The bold blue and red line represent ε_{II} and ω_{II} at the very center of the model, respectively with big dots indicating the external shear strain for which the inset figures are plotted. Model parameters are: L_y:h=2000:1 and η η_{eff}=100:1 (η: viscosity of the analogue medium).



[7] CONCLUSIONS:

- Heterogeneous strain occurs in all analogue simple shear rigs.
- Imperfect boundary conditions in the x-y-plane explain experiments with lubricated x- and y-walls.
- Imperfect boundary conditions in the z-direction explain experiments with lubricated lid or base plate.

TAKE HOME MESSAGES

- Error in strain is relatively small ⇨ Internal angles might be (almost) correct
- Error in rotation is relatively large ⇨ Angles to external reference line might be very inaccurate